Quantum Fields in a Big Crunch/Big Bang Spacetime

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Motivation

- **Ekpyrotic/Cyclic** models (based on **Heterotic M-theory**) provide an alternative picture of the origin of density perturbations in the universe.
- 4D Big-Bang=Collision of two higher dimensional Branes. Brane Ripples $\longrightarrow$ density pertubations.
- *Version 1: Collision of bulk and boundary brane.*
- *Version 2: Collision of two boundary branes.*

  Here assume branes pass through one another (familiar from BPS solitions in other contexts).
- **Ekpyrotic models can potentially solve flatness, homogeneity and isotropy problems without the need for inflation.**
- Matching of perturbations is a subject of much debate: Still unsolved!

Questions?

- Can the universe pass from a collapsing phase to an expanding phase?
- How do we match cosmological perturbations?
- What physics goes on when branes collide?
- Is late time physics dependent on what happens at the singularity?
- Can we describe brane collision within **String Theory** or **M-theory**?
- Can we model String description within a low-energy effective model?

Cosmo-2002

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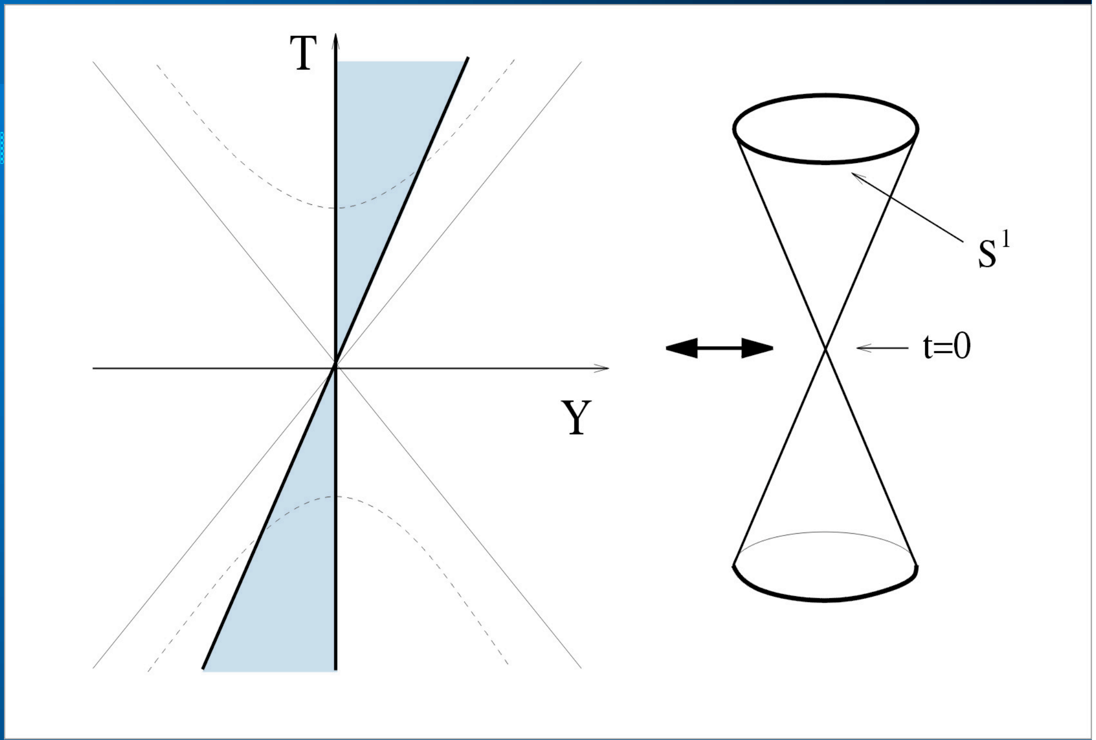
Compactified Milne spacetime

- Simple example of spacetime with cosmological singularity – compactified **Milne** (solution of **11D supergravity**):

$$ds^2 = -dt^2 + (H_5 t)^2 dy^2 + d\vec{x}^2$$

- Single extra dimension (y) shrinks to zero size and then re-expands at the same rate.
- Locally describes spacetime between two colliding branes.
- Identify y->y+L, equivalent to a boost, □= $X^\pm \to e^{\pm H_5 L} X^\pm$ in Minkowski co-ordinates: $X^\pm = T \pm Y$.
- Spacetime equivalent to region of Lorentzian Orbifold: **Minkowski/Boost**.
- Further orbifolding by Z_2 (y->L-y) gives spacetime between two colliding tensionless 'end of the world' branes, located at fixed points.



On the left is 2d Minkowski, dashed lines are surfaces of constant t, parametrised by co-ordinate y. Indentifying y->y+L compactifies space to produce cone on right. Further orbifolding circles by by Z_2 (y->L-y), two fixed points of the by Z_2 are two tensionless branes which collide and pass through one another at t=0

Matching of Quantum Fields: 3 approaches

In String point of view, String coupling vanishes, but the String geometry has a cosmological singularity.

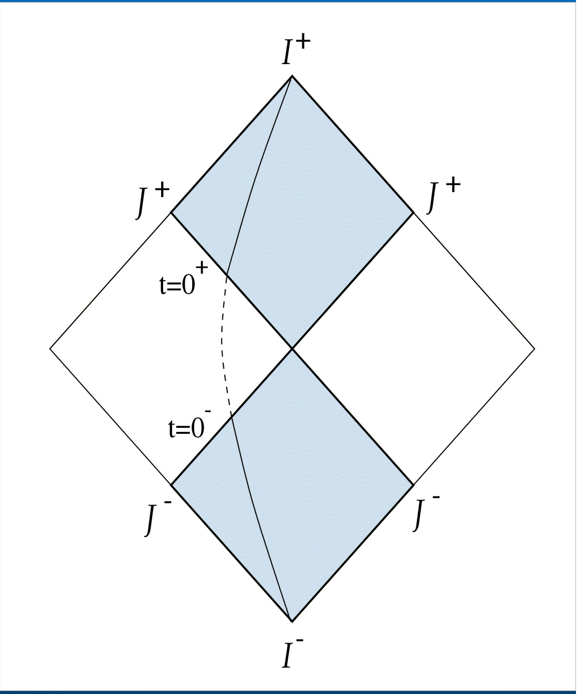
*Since the spacetime is singular (not **Hausdorff**) at t=0, Cauchy evolution breaks down. How can we match Quantum Fields across the Big Crunch/Bang Transition?*

Three methods:

- Determine matching from embedding in Mink. orbifolded by boost, □
- Analytically continue in complex plane to step around singularity
- Regularize spacetime and find minimal counterterms needed to cancel dependence on cutoff.

1. Embedding in Minkowski

- Compactified Milne is equivalent to a subspace of Lorentzian Orbifold, Mink/□.
- Natural states on Mink/□ form representation of group Poincare/□.
- Additional Rindler wedges contain **Closed Timelike Curves**.
- Restricting resulting states to Milne region determines a unique matching from collapsing to expanding phase.



Matching of Quantum Fields (cont.)

2. Analytic Continuation

- Singularity manifests itself as singular coefficients in differential equations for scalar field.
- Analytically continuing into complex plane allows us to step around the singularity.
- **Positive (Negative) frequency** wavefunctions analytic in lower (upper) half plane.
- Analogous to **'no-boundary'/Euclidean** methods .
- Similar to analytic continuation from Milne wedge to Rindler wedge implicit in the Mink. embedding.

3. Regularize and Renormalize

- Regulate singularity with cutoff, e.g.

$$ds^2 = -dt^2 + ((H_5 t)^2 + \epsilon^2)dy^2 + d\vec{x}^2$$

- Naïve answer is divergent as cutoff tends to zero, gives non-zero particle creation on all length scales leading to UV divergences.
- Generic behaviour independent of type of regularization.
- Solution: **Renormalize with a local Bogoliubov transformation**.
- Fixed uniquely by requiring vacuum to remain **Hadamard** and **Time reversal** invariant.

S-Matrix and Including Interactions

Three methods give unique answer for S-matrix |**in>=|out>.**

- Incoming adiabatic vacuum $\longrightarrow$ Outgoing adiabatic vacuum.

No particle creation on compactified Milne spacetime.

- Implies no *backreaction* problem at 1-loop.

Interactions

- Non-zero tree level amplitudes for creation of particles from vacuum due to absence of time translation invariance.
- **Interactions are computed by only integrating over physical spacetime** (discard Rindler wedges containing CTCs).
- Particle production for fixed external momenta is **finite at tree level**.
- Amplitudes fall off exponentially for large ky but only as power for large kx.
- Amplitudes exhibit **infrared divergences** – solved by more realistic cosmological spacetime (similar to IR problems in pure de Sitter).
- As brane velocity tends to zero, amplitudes for KK modes die exponentially.
- Amplitudes in general too 'hard' (slow fall off for large momenta), anticipate string theory will lead to 'softer' amplitudes.

Conclusions and Future Work.

Conclusions

- We have developed 3 methods giving a consistent and unambiguous answer for matching scalar field theory across a simple **Big Crunch/Big Bang transition** maintaining **Unitarity**.
- We have explicitly evaluated certain tree level scattering amplitudes finding **finite answers for fixed external momenta**.
- Integrated density also found to be finite for specific dilaton dependence of interaction coupling constants.
- We anticipate that **UV divergences** of loop diagrams will be **no worse** than in Minkowski spacetime because vacuum preserves the **Hadamard** condition.

Future Work

- Work is currently under way to see how these ideas may be used to determine the matching of the **full gravitational perturbations**, including the case where the brane collision is **inelastic (Tolley and Turok).**
- *Central issue is to understand how to describe brane collision in M-theory in order to finally answer question of how to match cosmological perturbations.*
- Much work needs to be done on understanding **Cosmological Singularities** in **String/M theory**!